# FLOW OF A NONLINEAR-VISCOPLASTIC FLUID BETWEEN THE TWO COAXIAL CYLINDERS OF A VISCOMETER FOR A STUDY OF THE MAGNETHORHEOLOGICAL EFFECT

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UDC 532.137.4:538.6

A relation is established between the dynamic and the kinematic characteristics of nonlinear-viscoplastic dispersions, applicable to an analysis of the magnetorheological effect.

It has been shown in [1] that the rheological properties of nonlinear-viscoplastic disperse fluids are generally rather well described by the rheological equation of state

$$\tau^{\frac{1}{n}} = \tau_0^{\frac{1}{n}} + (\eta\dot{\gamma})^{\frac{1}{m}}. \tag{1}$$

Experiments performed at the laboratories of the Institute of Heat and Mass Transfer, Academy of Sciences of the ByelSSR have established the applicability of the rheological equation (1) to nonconducting disperse systems moving in transverse electric and magnetic fields [2]. The nonlinearity of the flow curve is in most important practical cases characterized by the triparametric equation which follows from (1) with m = n, i.e., for uniaxial flow we have

$$\tau^{\frac{1}{m}} = \tau_0^{\frac{1}{m}} + (\eta\dot{\gamma})^{\frac{1}{m}}. \tag{2}$$

Rheometric tests on various dispersion systems are most often performed in rotating devices of the cylinder—cylinder, disc—disc, cone—plane, cone—cone, hemisphere—plane, or similar type. The theory of such devices has been rather thoroughly studied, concerning anomalous—viscous systems (nonplastic dispersions) which obey the power-law rheological equation of state.

We will consider a fluid which behaves mechanically according to Eq. (1) under simple shear in a viscometer between two coaxial cylinders.

We start out with the more general notation

$$p_{ik} = \left[\left(\frac{\tau_0}{h}\right)^{\overline{m}} + (\eta h)^{\frac{1-m}{m}}\right]\dot{e}_{ik}. \tag{3}$$

In subsequent calculations we assume that $H/\delta \gg 1$. The problem of estimating the error due to the bottom effect requires a careful analysis. The assumption is valid under practical conditions. Consequently,

$$v_r = v_z = 0 \text{ and } v_\varphi = f(r). \tag{4}$$

The differential equation yields

$$\tau = \frac{c}{r^2}. \tag{5}$$

Let the outer cylinder be stationary and the inner cylinder revolve. The torque applied to the inner cylinder is usually known. Then disregarding the drag torque at the bottom surface of the inner cylinder,

Byelorussian Polytechnic Institute, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 23, No. 3, pp. 482-485, September, 1972. Original article submitted February 28, 1972.

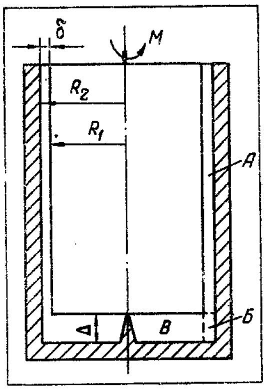


Fig. 1

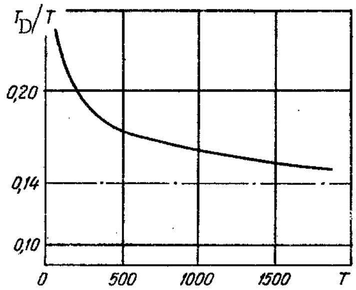


Fig. 2

Fig. 1. Schematic diagram of a device consisting of coaxial cylinders.

Fig. 2. Calculation of the bottom effect.

one can determine the constant c:

$$c = \frac{M}{2\pi H}. \tag{6}$$

Under these conditions and with m = n, the inequality $\dot{e}_{r\varphi} < 0$ must be satisfied,

$$\dot{e}_{r\varphi} = -\frac{1}{\eta^m}\left[\frac{c^{\frac{1}{m}}}{r^{\frac{2}{m}}} - \tau_0^{\frac{1}{m}}\right]^m. \tag{7}$$

Expressing $\dot{e}_{r\varphi}$ in cylindrical coordinates, we obtain after integration

$$\frac{v_\varphi}{r} = \omega_0 - \frac{1}{\eta^m}\int_{R_1}^{r}\frac{1}{r}\left(\frac{c^{\frac{1}{m}}}{r^{\frac{2}{m}}} - \tau_0^{\frac{1}{m}}\right)^m dr. \tag{8}$$

If torque M is larger than $2\pi R_2^2 H\tau_0$, then shear flow must occur within the entire region $R_1 \le r \le R_2$. In order to determine $\omega_0$, we use the condition of fluid adhesion to the outer cylinder $v_\varphi(R_2) = 0$. Then

$$\omega_0 = \frac{1}{\eta^m}\int_{R_1}^{R_2}\frac{1}{r}\left(\frac{c^{\frac{1}{m}}}{r^{\frac{2}{m}}} - \tau_0^{\frac{1}{m}}\right)^m dr. \tag{9}$$

When $M < 2\pi R_2^2 H\tau_0$, then the region of shear flow does not extend to the outer cylinder and the outside radius $R_2^1$ of this region is

$$R_2^1 = \sqrt{\frac{M}{2\pi H\tau_0}}, \tag{10}$$

while the angular velocity of the inner cylinder is

$$\omega_0 = \frac{1}{\eta^m}\int_{R_1}^{R_2^1}\frac{1}{r}\left(\frac{c^{\frac{1}{m}}}{r^{\frac{2}{m}}} - \tau_0^{\frac{1}{m}}\right)^m dr. \tag{11}$$

For a rough estimate of the error incurred by disregarding the bottom effect, we determine the drag torque at the lower end surface of the inner cylinder revolving at the speed $\omega_0$. The closeness of the gap between the flat end surfaces of both cylinders allows us to assume that $v_r = v_z = 0$ and $v_\varphi = \Phi(r, z)$,

$$\dot{e}_{r\varphi} = \frac{\partial v_\varphi}{\partial r} - \frac{v_\varphi}{r} = 0. \tag{12}$$

A solution accounting for the contiguity of regions A, B, and C (Fig. 1) can be obtained by the variational method, for example.

With the assumption made here, we will find the circumferential velocity $v_\varphi$ as

$$v_\varphi = \frac{\omega_0 r z}{\Delta} \tag{13}$$

the azimuthal component of the shear stress

$$\tau_{\varphi z} = \left[ \tau_0^{\frac{1}{m}} + \eta^{\frac{1}{m}} \left( \frac{\omega_0 r}{\Delta} \right)^{\frac{1}{m}} \right]^m , \tag{14}$$

and the drag torque at the end surface of the inner cylinder:

$$M_D = 2\pi \int_0^{R_1} r^2 \left[ \tau_0^{\frac{1}{m}} + \eta^{\frac{1}{m}} \left( \frac{\omega_0 r}{\Delta} \right)^{\frac{1}{m}} \right]^m dr.$$

Our measurements have yielded the following values: $\tau_0 = 20\ \mathrm{N/m^2}$, $\eta = 0.05\ \mathrm{N \cdot sec/m^2}$, $m = 2$ with $R_1 = 0.015$ m, $R_2 = 0.016$ m, $\delta = 0.001$ m, $\Delta = 0.008$ m, and $H = 0.06$ m. For these conditions we have

$$M_D = \frac{2}{3} \pi \tau_0 R_1^3 + \frac{8}{7} \pi \sqrt{\frac{\tau_0 \eta \omega_0}{\Delta}} R_1^{\frac{7}{2}} + \frac{\pi}{2} \cdot \frac{\eta \omega_0}{\Delta} R_1^4.$$

With the dimensionless parameters $\xi = \Delta/R_1$ and $T = M/\pi\tau_0 R_1^3$,

$$T_D = \frac{2}{3} + \frac{8}{7} \left[ \frac{TR_1^3}{4H\xi\eta} \left( \frac{1}{R_1^2} - \frac{1}{R_2^2} \right) + \frac{R_1}{\xi\eta} \sqrt{\frac{2TR_1}{H}} \left( \frac{1}{R_2} - \frac{1}{R_1} \right) + \frac{1}{\xi\eta} \ln \frac{R_2}{R_1} \right]^{\frac{1}{2}} + \frac{1}{2} \left[ \frac{TR_1^3}{4H\xi\eta} \left( \frac{1}{R_1^2} - \frac{1}{R_2^2} \right) + \frac{R_1}{\xi\eta} \sqrt{\frac{2TR_1}{H}} \left( \frac{1}{R_2} - \frac{1}{R_1} \right) + \frac{1}{\xi\eta} \ln \frac{R_2}{R_1} \right].$$

The ratio $T_D/T$ has been plotted in Fig. 2 as a function of the total dimensionless torque T. The curve has the shape of a hyperbola with an asymptote $(T/T_D)_{asymp} = 0.14$. Thus, by simple calculations, we can find the optimum dimensionless ratio of useful torque to stray torque $T_L/T_D = 6$.

## NOTATION

| | |
|---|---|
| $R_1$, $R_2$ | are the radius of inner and of outer cylinder; |
| $R_2'$ | is the outside radius of shear region; |
| $R_1 \le r \le R_2$ | |
| H | is the height of inner cylinder; |
| $\Delta$ | is the gap between bottom of inner and bottom of outer cylinder; |
| $\delta = R_2 - R_1$; | |
| $\omega_0$ | is the angular velocity of inner cylinder; |
| $\dot{\gamma}$ | is the shear rate in uniaxial flow; |
| $M_D$, $T_D$ | are the dimensional and dimensionless drag torque at the bottom of inner cylinder; |
| M, T | are the dimensional and dimensionless torque applied to inner cylinder; |
| $T_L$ | is the dimensionless torque at the lateral surface of inner cylinder; |
| $\tau_0$, $\eta$, m, n | are the rheological parameters of the system; |
| $\tau$ | is the tangential shear stress; |
| $\tau_{\varphi z}$ | is the azimuthal component of shear stress in the gap; |
| $p_{ik}$ | are the components of stress tensor; |
| $\dot{e}_{ik}$ | is the strain rate tensor; |
| h | is the strain rate density; |
| $v_z$, $v_r$, $v_\varphi$ | are the velocity components in cylindrical coordinates; |
| c | is the integration constant. |

## LITERATURE CITED

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